NCTM Principles and Standards for Mathematically Talented Students

Linda J. Deal and Michael G. Wismer

The *Principles and Standards for School Mathematics* published in 2000 by the National Council of Teachers of Mathematics (NCTM) created a vision of mathematical concepts and processes to establish core educational guidelines for instruction from grades K to 12. Although this document does not mention talented students explicitly, it does acknowledge that students are not at the same ability levels. The overall plan does emphasize higher level thinking, problem solving, and communication skills that were traditionally advocated for gifted learners but the implementation of this vision continues to fall short when serving mathematical talent. With the advent of No Child Left Behind (NCLB, 2001), less able math students are provided with support and alternative instruction to meet the proposed standards. Little has been done to identify and serve highly capable students until the high school level. The purpose of this paper is to introduce an understanding of talented mathematical students and to learn how the NCTM *Principles and Standards* can be modified to provide support for these students. Although acceleration may help to match the existing curriculum with student abilities, additional adaptations and modifications of the NCTM *Principles and Standards* are necessary to meet each student's rate of acquisition, asynchronous development, intense focus, and complexity of thought (Assouline & Lupkowski-Shoplik, 2003, p. 185).

Understanding Gifted and Talented Individuals

Despite the high-level principles and standards presented by NCTM, specific characteristics of talented individuals are not recognized. Although it is difficult to generalize about students who are gifted, as a group they tend to exhibit the following traits (Berger, 1991; Johnson, 2000):

- fast rate of acquisition,
- high rate of retention of material learned,
- complexity of thought,
- asynchronous development, and
- curiosity.

A fast rate of acquisition indicates that the pacing in a traditional classroom may be too slow for a typical gifted child. Although most students require many repetitions to transfer new information from their short-term memory to their long-term memory, talented students may learn information precociously and rapidly (Chuska, 2004; Piirto, 2007). Repetitions and ongoing drills may lead to a frustrating situation that eliminates students' excitement for math.

Most math curricula's scope and sequence involve ongoing review of ideas presented previously. Because gifted students have a high rate of retention, this review is often unnecessary. Indeed, many students may skip the first three chapters of the math text if pretesting and a brief overview of items missed on the preassessment are available.

Talented individuals think with a complexity that creates new ideas and deepens meaning of ideas. They transfer ideas and patterns to unusual situations, make connections between unrelated topics, and have an intense curiosity to question and go beyond

what has been introduced (Johnson, 2000). Unfortunately, because of the sequential approach that most math textbooks use, there is little time to honor and explore exciting tangents that many gifted students wish to follow.

Gifted children often experience asynchronous development. Although all children develop at different rates in their physical, intellectual, social, and emotional growth, the rate of change for gifted individuals is typically far more extreme (Roedell, 2000). For example, a preschool child could tell time with an analog clock perfectly. One day, his watch broke. He cried uncontrollably. His teacher offered him her watch and tried to comfort him. When he calmed down enough, he explained to his teacher that if his watch was broken, his mother wouldn't know when to pick him up, and until it was fixed, he wouldn't see her. His intellectual understanding of a skill and his emotional development were not synchronous. This trait may follow gifted children throughout their teenage development. In addition, their vocabulary level frequently makes others feel as though they are small adults in all areas of development.

Understanding Mathematically Talented Students

Mathematically talented students are often difficult to recognize. Although curriculum-based assessments (CBAs) indicate ability with information presented, they do not always identify mathematical talent. CBAs only indicate mastery of specific skills to which students have been exposed. The reasoning ability of the mathematically talented student may be 2 or more years beyond the current curriculum. Most CBAs usually are based on computational ability and less on reasoning skill (Mann, 2006). Although mathematically talented students may do well on these assessments, they do not touch on the research-based behaviors that yield clues to identifying this talent. Although many lists of mathematical talent characteristics exist (Rotigel & Fello, 2004), a summary includes the following:

- keen awareness and curiosity about numbers;
- fast rate of acquisition specifically geared toward understanding and applying mathematical ideas;
- ability to work and think about abstract mathematical patterns and relationships;
- possession of analytical, deductive, and inductive reasoning skills without exposure to these abilities;
- ability to use flexible and creative thinking, rather than sequential or standard forms of reasoning, to approach mathematical problems; and
- ability to transfer mathematical reasoning to new and untaught situations.

It should be noted that computational skill and accuracy are not mentioned on talent checklists. Yet, the majority of CBAs evaluate math calculation skills rather than math reasoning ability. Students leave school with adequate computational ability but lack the ability to apply computations in meaningful ways (Mann, 2006).

Another drawback to an assessment approach for identifying talent is the ceiling effect of tests. Even with standardized, normed assessments, there are rarely enough out-of-level questions on a test to appropriately determine the upper limits of ability. According to Assouline and Lupkowski-Shoplik (2003), out-of-grade-level (2 years above) assessment is recommended as a starting point to indicate mathematical talent.

Mathematical Principles and Mathematically Talented Individuals

The Equity Principle

As outlined by NCTM (2000), the Equity Principle states:

> *Excellence in mathematics education requires equity—high expectations and strong support for all students* [italics in original]. . . . Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed . . . for all students. . . . All students need access each year to a coherent, challenging mathematics curriculum . . . Equity requires high expectations and worthwhile opportunities for all. (p. 12)

If the Equity Principle were honored in traditional classrooms, there would be no need for Gifted Individualized Educational Plans (GIEP) in math. If "reasonable and appropriate accommodations" were made for talented students, and if they were provided with a coherent and challenging curriculum, talented math students would more than excel. In reality, few teachers recognize true math talent and few teachers know how to make accommodations for these students. To be fair to teachers, though, teacher training rarely includes information about talented students, and although textbooks provide enrichment activities, they rarely involve the rigor that these students demand.

According to Maker (2004), reasonable accommodations for these students might include whole- or multiple-year acceleration, enrichment, change in pacing the curriculum, compacting, and eliminating the repetitive nature of the math curriculum. Above-grade-level assessments can be used to determine the need for acceleration. Even if acceleration is indicated and the placement is made, it should be noted that the pacing in the upper grade class may still be slow and redundant to the bright student. Pretesting with a curriculum-based chapter test can indicate sections of the text that this student may skip (compacting) and the extra time can be used for parallel, in-depth study and application of the skills being taught. If the preassessment indicates that a student needs to be a part of every lesson, then his or her daily work should be modified as needed. If a student can do the hardest problems on the review sheets (usually the last several questions), then the constant repetition of similar problems could be substituted with higher order, more rigorous mathematical thinking activities and applications.

Reasonable accommodations do not include helping or tutoring other students on a daily basis (Assouline & Lupkowski-Shoplik, 2003). Although this may occasionally help the talented math student with communication skills in math, a steady diet of this does not invite the excitement and passion for math that we hope to maintain. Nor do reasonable accommodations mean that these students must complete all of the assignments that the regular class works on—and then do special problems on top of that work (Johnson, 2000). The special problems should be substituted work or the students will quickly realize that they are doing much more work than other students (Davidson Institute for Talent Development, 2003).

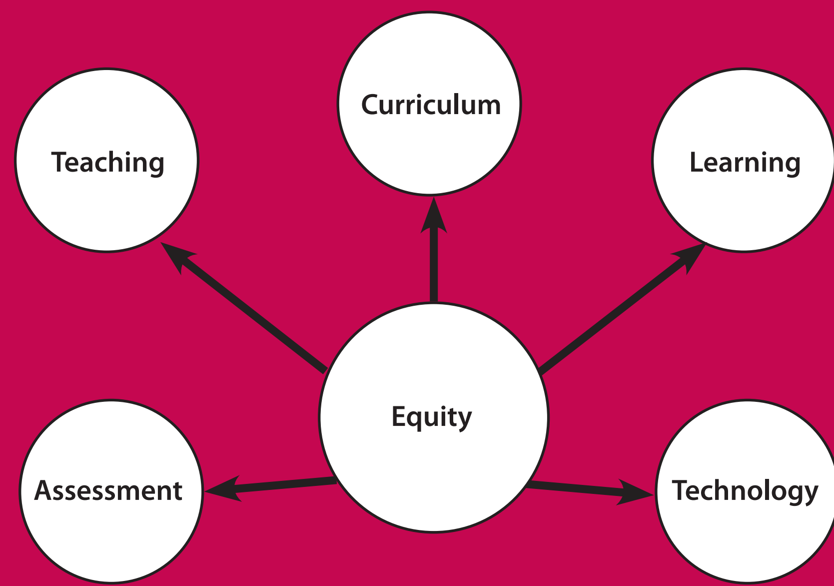

***Figure 1*. The relationship of the Equity Principle to the other five NCTM principles.**

Basically, reasonable accommodations would follow any differentiation provided for all students, whether remedial or talented. The content can be changed by offering more challenging problems, mathematical reasoning, higher grade-level work, or enriched studies of topics like topology, tessellations, and history of mathematics. According to Sousa (2009), the process can be changed by providing problems with multiple answers or searching for new patterns, and the products can be changed by new applications, transferring concepts to other subject situations, changing strategies, or the use of reflection and imagination.

Whatever the approach used, it should uphold the principle of high expectations and rigor with ongoing resources and support from the teacher. If we truly apply the Equity Principle to math instruction, then the other five principles that follow will naturally fall into place for the mathematically talented student. The Equity Principle is the basic hub for the other principles as illustrated in the diagram shown in Figure 1.

The Curriculum Principle

According to NCTM (2000), "*curriculum is more than a collection of activities. It must be coherent, focused on important mathematics, and well articulated across the grades* [italics in original]" (p. 14). The scope of mathematical textbooks presents a sequential, spiraling curriculum that is often repetitive in nature. For example, multiplication tables are presented as early as grade 3 and continue through grade 6. Talented math students can frequently pretest out of the first three chapters of the math text with a 1-week review. Teachers can accommodate these students in a variety of ways. During this time, they can provide alternate, in-depth, but parallel, lessons and challenge for these students or they can provide creative activities and enrichment topics that encourage breadth of mathematical thinking. These topics should, however, be cho-

Table 1
Positive and Negative Aspects of Standards

Positive aspects of standards	Negative aspects of standards
Common coherent elements	Lack of piloting and revision
Consensually developed by the discipline	Lack of translation models
Allow for differentiation and creativity	Lack of consonant assessment of these strategies
Teacher latitude in implementation	Lack of consistent application of research
Represent "vision" of adult competencies	Politicized and polarized

sen carefully so that instruction continues to be focused and coherent—not random, isolated skills or topics.

The goal of mathematics instruction for these talented students should not only be to build a repertoire of mathematical skills, but also to encourage mathematical habits of mind. These habits include creativity, tenacity, skepticism, and collaboration (VanTassel-Baska & Little, 2003). Open-ended explorations help encourage this type of reasoning. Problems with just one process and one correct solution do not encourage this type of thinking. As Johnson (2000) found, talented math students should be invited to think about problems that have not been asked, wonder if things are always true, and sort through messy real-life problems that do not have obvious algorithms to follow.

VanTassel-Baska (2009) presented the positive and negative aspects of using standards at the National Association for Gifted Children conference in St. Louis, MO. They are reflected in Table 1.

VanTassel-Baska (2000) emphasized that, while curriculum standards are needed, they need to be modified for gifted individuals. Although most of the math standards are based in best-use practices and there are many books and resources available on how to implement the standards, some of the negative aspects apply to the field of talented math students. Gifted students need to know the standard curriculum but little revision is made in the *way* these standards are presented to talented math students. The consensus of what skills students need to learn is noteworthy, but how do these skills get translated for students with mathematical talent? Differentiation and creativity are both inherent in the principles, but how can a teacher assess when to use a differentiated approach and if progress in creative mathematical thinking has occurred? Teachers appropriately have been provided with latitude to select teaching techniques; but they may not have been provided with information on how a talented, mathematical mind approaches skills and problems. In addition, in their training, they most likely have not been given research-based information on talented thinkers. The *2008–2009 State of the Nation in Gifted Education* (National Association for Gifted Children [NAGC], 2009) report documented the need for teacher training when working with gifted students. The survey found that 40 states indicated a need for undergraduate training in gifted education, 40 states indicated that regular education teachers need training, and 20 states indicated that they have low to no standards for licensure to teach gifted students.

Traditional methods do not place much emphasis on creative applications in exploring math processes instead of just moving at a rapid pace or learning algorithms. In the traditional classroom, the methods used often teach closed problems with predetermined answers (Mann, 2006). The current focus with NCLB (2001) has emphasized speed, accuracy, rules, convergent thinking, and algorithms. Talent development in math requires encouraging habits of mind that go beyond these basic skills. Talent development needs to reinforce creative thinking (Mann, 2006). The right answer/wrong answer approach inhibits the growth of independent mathematical reasoning and denies opportunities for independent thinking, originality, and exploration. All of these are higher order thinking skills. Denying creative math skills can delay—and even prevent—potential development for later advancement of math applications and theory (Mann, 2006).

Teachers need to help students look beyond "wrong" answers and explore the creative applications of math reasoning that students may have used to arrive at their answers. This acceptance will help students explore, question, and interpret their reasoning. Accuracy in math is important but accuracy without understanding is of minimal use. The right answer to the wrong problem is potentially harmful on a construction site! There will also be little use in the future for individuals who are trained to do tedious computations as technology advances.

There are techniques to aid with creative approaches. These can include, but are not limited to:

- provide open-ended problems with a range of alternative solution methods to discuss and develop;
- reach beyond the familiar, obvious problem by probing deeper into relationships between factors in a problem and the structure of the solutions; and

The following sites are based on the NCTM standards and provide activities or projects for mathematically talented students. Some sites provide a basis for investigations, research, and creative problem solving discussions.

Mathwire.com
http://mathwire.com
Teacher-tested activities and worksheets can be downloaded for free at this standards-based site. The site can be searched in different formats by standards and new additions, as well as alphabetically, and it includes a blogging option.

ThinkQuest
http://www.thinkquest.org/en/projects/index.html
At this site, you can join other projects already under way or design original math projects and have others join in the exploration.

Mathematics in Movies
http://www.math.harvard.edu/~knill/mathmovies
This site features movie clips that show math in action. This interesting site can generate discussion and research based on popular movies and television shows. It includes everything from *Shrek the Third* to *Apocalypse Now* to *Phantom Tollbooth*. Flash versions and QuickTime clips can be downloaded for view. *Note: The movie choices on this site should be prescreened for appropriate language.*

U.S. Food and Drug Administration (FDA)
http://www.fda.gov/Food/GuidanceComplianceRegulatoryInformation/GuidanceDocuments/Sanitation/ucm056174.htm
At first glance this site may not appear very interesting, but it provides many hours of investigation on foodstuffs and the contaminants that the FDA allows. Using data provided, students must apply ratios and juggle different measurements to determine how many bugs are in peanut butter, how many rotten tomatoes are in ketchup, and the like.

Scratch
http://scratch.mit.edu
This free site provides an introduction to the logic of computer programming. Students can create their own games and begin to learn the basics of programming skills with help from online educators and a design studio. Students can try many of the games others have created and may post their own creations.

Gapminder
http://www.gapminder.org
Gapminder shows the world's most important trends with animated graphs that plot up to four different parameters at once. The fascinating progress on the graphs can provide the basis for stimulating discussions on the statistical changes in world population, environment, and agriculture.

Illuminations
http://illuminations.nctm.org
This site contains a bonanza of information provided by NCTM that is both standards based and aligned. It provides activities, lesson plans, mathematical investigations, and links to other activity sites that will stimulate interest and challenge mathematical reasoning skills. The lesson plan page is particularly helpful because it contains recommended grade-level ranges for topics.

XP Math
http://www.xpmath.com
This site provides math homework help, games to play, problems of the week, and math challenges.

***Figure 2.* Math websites.**

- provide situations where students develop the problems from other research or data provided.

Two interesting activities that use the last strategy can be applied across all grade levels and cross over into media and technology approaches. The Federal Food and Drug Administration maintains a site that lists the amount of contaminants allowable in foods that are purchased and eaten. Students may establish ratios and convert measurements to determine the amount of undesirable pollutants that are found in favorite foods. They create their own questions and problems based on data provided by the government. A second site, Mathematics in Movies, encourages students to develop problems around video clips they view on television. Students may even build their own clip collection with math problems and share them on a class website. See Figure 2 for a list of websites.

Technology use is currently receiving a lot of emphasis to address the needs of talented students (Rotigel & Fello, 2004). Many districts have tried to solve the problem of pacing and acceleration by using commercial online math programs. Frequently, the student ends up working in isolation with the computer and does not have opportunities for interaction about the content with the teacher and with peers. Additionally, although these programs may be well written, they rarely fit well with the focus and coherence of the district scope and sequence. If a student is provided with this type of instruction for one year and then returns to a traditional, advanced class the following year, he or she is at a disadvantage because key concepts may have been missed. Although the novelty of working on the computer is enticing at the start, with time, these students miss the ongoing communication on which problem solving is based. Blended courses—ones that combine working on the computer and instruction from a teacher—may solve the communication issue, but the

curriculum itself may be out of step with the cross-grade-level articulation and sequence. Loveless and Coughlan (2004) found that if the student returns to the traditional curriculum for later courses, there may be gaps in specific skill abilities.

The advantage of some of these programs is that students may be self-paced as they work through the curriculum. For example, Renaissance Learning's Accelerated Math program (http://www.renlearn.com/am) provides flexible pacing, creates individualized practice worksheets based on assessment, and generates diagnostic reports to help instructors pinpoint student difficulties. This can help make acceleration and compacting more manageable in a traditional classroom, because the teacher can track student performance and offer support when needed without time-consuming monitoring of student work. It is important to note that this type of option may require district resources (expenses, computers, and technology support) that may not be readily accessible (St. Cyr, 2004).

The Teaching Principle

According to NCTM (2000), the Teaching Principle "*requires understanding what students know and need to learn and then challenging and supporting them to learn it well* [italics in original]" (p. 16).

Note that the first part of this principle refers to understanding what the students already know. Usually, at the introduction of a new concept, some form of preassessment in the classroom is administered to assess whether students have the prior knowledge basis on which to build. Talented math students require much more than this check. A teacher must gain an understanding of whether the students already know the new concept or topics being presented over the next few chapters, or even the entire book, to grasp the true ability placement of the individual. Preassessments of this type can be formal, informal, or hands-on. They should allow for differences in understanding, creativity, and accomplishment.

Once the appropriate knowledge base is established, differentiated math instruction should be used within the class. Differentiation is not just doing a different assignment; it is modifying the level of work to accommodate the individual needs of students. Tiered lessons is one type of differentiation strategy. Possible tiered thinking skills include the following choices in study: provide options for analysis, choose different problem-solving strategies, apply the skill to new areas, or use unusual creative applications. Examples of tiered mathematical lessons can be found in books by Edward Zaccaro; directions for writing layered lessons can be found in *Generating Standards-Based Lessons* by John Lester (2007) and *Differentiating Instruction in the Regular Classroom* by Diane Heacox (2002). Students may also be cluster grouped with other students that have the same knowledge base for that chapter. Johnson (2000) offered these additional suggestions for teaching talented math students:

- Use a variety of text sources with gifted students. Math textbooks are used about 80% of the time in math instruction (Lockwood, 1992). However, several series that hold promise for gifted learners have been developed (e.g., the Math Connects series by McGraw-Hill). These books emphasize constructivist learning and include concepts beyond the basics. *No single text will adequately meet the needs of these learners.*
- Provide flexible pacing opportunities within the classroom: teach the hardest item first; consider whole-chapter replacement work and daily enrichment.
- Extend replacement work (enrichment) beyond the normal curriculum. Mathematical recreation activities, such as puzzles, logic activities, and strategy games, should be included. Note that this is *in replacement of* and not *in addition to* the normal class work.
- Emphasize inquiry-based, discovery learning approaches through open-ended problems with multiple solutions or multiple paths to solutions. Talented math students may discover more than one may think is possible.
- Use many higher order questions to justify and discuss problems. Ask "Why?" and "What if . . .?"
- Expect higher level products: writing proofs, projects, and solutions to challenge problems.
- Provide opportunities to participate in contests and competitions such as Math Olympiads (grades 4–6), Math Counts (grades 7–8), American Mathematics Competitions, American Junior High School Mathematics Exam, and others. Even if accelerated students do not meet the grade-level/age-level requirements for the competition, the practice problems from past events provide excellent challenge.
- Invite both male and female multicultural speakers to class or contact them by e-mail to explain how math has opened doors in their professions and careers.
- Provide useful concrete experiences. Although talented learners may be capable of abstract thought, they still benefit from the use of manipulatives and hands-on activities.

The most important of all of these suggestions is to become aware of the need to acknowledge the difference in

the pacing of lessons. Even accelerated students may need daily enrichment and whole-chapter replacement work. It is best to be flexible among all three: acceleration, replacement enrichment, and daily enrichment.

Classroom teachers and school districts share the responsibility of addressing the teaching needs for these students. Teachers need training and support to learn about the mathematically talented, as well as the mathematically remedial student. Teachers of math at all levels need to have a strong background in mathematics content so that they can take advantage of the teachable moments of advanced enlightenment these students see. A coordinated curriculum plan for enrichment and advancement needs to be in place so that the experiences presented are not duplicated nor interrupted. The district needs to provide a support system that includes resource books, technology, and community resources (Johnson, 2000).

A coordinated curriculum plan is particularly needed for students who have been accelerated. Curriculum becomes an issue when students are cross-grade leveled only in math and not in their other content areas. When these students reach transition years (from elementary to middle school and middle school to high school), a different delivery model for instruction is critical (St. Cyr, 2004). There have been different options considered. Some districts have the student start the school day at the higher level school for math only and then transport the student to the traditional setting once the class is over. Another option is to use an online course for instruction. There are many well-known universities that offer these transition-level courses. When the students rejoin the traditional curriculum offered in the district, assessments can help determine if there are gaps between the university program and the district program (St. Cyr, 2004).

Penn Manor School District in Pennsylvania has taken several creative approaches with the new technology available. Depending on scheduling and the level of the math course, the accelerated students meet with the math teacher every other day in person. On the alternate days, the students and teacher iChat throughout the lessons. In this way, the teacher can service two or more schools and multiple accelerated students simultaneously. The students get to communicate with each other and are not doing their lessons in isolation. The alternate day, face-to-face teacher support, provides reinforcement and opportunity for less-structured instruction than is needed in the online class time. On days when the technology will not cooperate, a Moodle site is available with daily lessons, practice links, and assignments for the students not receiving the face-to-face instruction. This coordinated plan has benefited the students by providing for communication with mathematical peers while using the cohesive scope and sequence of the district curriculum with no educational gaps. The other approach used within this district is that a high school teacher moves to the elementary school and provides instruction for students who are radically accelerated more than 2 years.

The Learning Principle

According to NCTM (2000), "*Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge* [italics in original]" (p. 20).

In order to apply learning principles, teachers first must have an understanding of how the talented math mind learns. Krutetskii, a soviet researcher who investigated characteristics of mathematically talented children, suggested that there are three stages of mental activity to solve mathematical problems: gathering, processing, and retaining information (Pierce, 2002). He developed a diagnostic series of 26 experimental, extensive problems and completely described them in his book. From his work, Krutetskii contended that some people tend to interpret the world mathematically. These mathematical mindsets fell into three types of vision: (a) the verbal-logical, analytic, or mathematical abstract; (b) the verbal-pictorial or geometric mind; and (c) the harmonic mind, which has a combination of the abstract and the pictorial. The approaches of these mindsets are not clear-cut, and they do not always stand alone as isolated constructs. The thought processes overlap each other and also are not clear-cut. The verbal-logical learner will break down a process using a sequential, ordered approach to a problem. This is the method used most frequently within classrooms. It is "proof" type thinking that can explain what properties were used and why it was done that way. This style of thinking is typically thought of as a "left-brained approach." The pictorial or geometric mind thinks more visually. These students frequently just "see" the answer in their mind and may not be aware of how they made leaps to find it. They process problem solving in a different manner than the sequential-logical approach. The harmonic mind will pull from both sides of the brain and use both styles of thinking in different situations, as documented by x-rays of brains during problem-solving procedures.

Mathematically talented students may take an upside-down approach to the way typical students learn. They view calculations as tools for problem solving and grasp concepts and reasoning abilities far beyond their compu-

tational skill. They may not have the best grades in their class because computation itself may be cumbersome. But the excitement and approaches to reasoning through a problem far exceeds their computation drill and the computations will eventually follow the reasoning as the need arises.

Selecting alternate lesson plans will be key to this type of learning mind. Students will thrive on open-ended problems. This type of problem will also help to build persistence and tolerance for ambiguity, which is a mindset necessary for advanced mathematical processes.

Textbooks are staples of U.S. mathematical instruction. Teachers use them to plan lessons, and students take them home almost daily. Their impact usually facilitates the standards but may compromise mathematically talented students' learning. The number of pages a textbook devotes to a topic influences the amount of time the teacher will spend on that topic. It is best if teachers no longer simply cover the material. Rather, an approach that encourages questioning, conjecturing, problem formulation, and multiple strategies is key for talented students. The questions below, adapted from Reys, Reys and Chavez (2004), should be asked about a textbook to help determine how appropriate it is for the talented learning mind:

- What key mathematical ideas and reasoning in each content strand should be addressed?
- How does the content of the text align with these concepts?
- Do the activities provided challenge students to think or to simply follow drill patterns?
- Will the activities engage and challenge student thinking, communication, and transfer of ideas to new areas?
- Is there a focus on the "why" of mathematical thinking and problem solving?

The Assessment Principle

According to NCTM (2000), "*Assessment should support the learning of important mathematics and furnish useful information to both teachers and students* [italics in original]" (p. 22).

Notice that there are two parts to the statement above. First, assessments should support learning mathematics and, second, assessments should furnish useful information to both teachers and students. These ideas require different forms of assessment and will be addressed separately below.

Adapting Curriculum-Based Assessments for Current Content. Most mathematical assessment that is consistently used is found at the end of a chapter or a unit to determine if a student has mastered a specific skill (Mann, 2006). These curriculum-based assessments will be addressed first because that is the most common assessment reference.

In many textbooks, most of the practice problems at the end of each chapter focus on drill of the skill, and only a few of the problems are "word problems" that are based on real-life application of the skill presented. End-of-unit tests usually follow this same pattern. The majority of the questions on these assessments document that the student can do the calculation pattern presented in the chapter. The remainder of the test is then based on applying and using the skill. Reys et al. (2004) presented issues like these that express concerns about America's math textbooks. Most of the time, assessments (and practice) for talented math students should have the reverse emphasis. Only 20% of the assessment should be based on the drill of computation, and the remainder of the test should emphasize students' thinking and reasoning ability. This would be an easy accommodation to make for mathematically talented students and would tell far more about their understanding of the concepts presented.

Alternative assessments also should be considered for the mathematically talented. They should be provided with opportunities to journal about their thinking processes, consider choice with authentic projects (like scale drawings for a geometry lesson), or create their own problem to demonstrate their skill with the concept.

Curriculum-based assessments may be in the form of preassessments, formative assessment, or final assessments. Preassessments are the most useful with talented math students. Preassessing a unit or chapter can indicate which lessons—or even whole chapters—a student may miss. These sections can be replaced with alternative lessons that are more meaningful for that student. By keeping records of these assessments, teachers may look for students who consistently know 80% or more of a chapter before it is taught (Assouline & Lupkowski-Shoplik, 2003; VanTassel-Baska & Little, 2003). These students should be considered for acceleration. However, the first three chapters of a textbook should not be included in this analysis, because these chapters usually review the previous year's instruction. Acceleration may take place within the classroom through compaction; students can then be allowed to pursue independent projects and research on math-related topics (Johnsen, 2005). Acceleration may also take other forms such as grade skipping; cross-grade-level placement in math; simultaneous dual enrollment in high school and college courses; and college-level courses taken online or at a local university. Radical acceleration of more than 2 years should be considered for students with exceptional abilities (Colangelo, Assouline, & Gross, 2004).

Curriculum-based assessments are very limited in scope to help determine the true ability of mathematically talented students. These assessments only ask students to replicate procedures that have already been taught in class. Mathematical reasoning talent is beyond the daily instruction for computation. In order to determine the mathematical talent of a student, very different types of assessments need to be considered. Preassessment is key to matching instruction with student ability, but out-of-level assessments are necessary to determine how far their knowledge extends. Students need opportunities to take assessments that are 2–3 years beyond their grade-level assignment.

Assessing Information for Mathematical Reasoning Ability. To determine mathematical talent, an assessment should be nationally normed. This broad basis for comparison helps provide a database that will document the true ability of a student. Nationally normed tests that are often considered are IQ tests and achievement tests. These tests, however, rarely have enough questions focused on mathematical reasoning to give a true indication of the ceiling of ability nor do they emphasize abstract, algebraic reasoning. Refer to Table 2 for a comparison of assessments.

Table 2
Comparison of Assessment Strengths and Weaknesses

IQ Tests	Achievement Tests	Math Aptitude Tests
Help with identification but not sufficient math information	Often computation-oriented; provide little information on reasoning ability	Usually emphasize math reasoning more than computation skill
One overall score provided for several components of mathematical ability	Seldom have enough difficult problems to measure upper limits	Similar to achievement test limitations
May be high in verbal skills but not math skills	Do not measure qualitative differences in math thinking	May measure some differences in qualitative thinking

Again, when searching for mathematical talent, out-of-grade-level norms should be considered to evaluate the raw scores collected. At the elementary level, Johns Hopkins University has developed several quality math aptitude tests to assist with this. At the middle school and high school levels, the SAT or similar types assessments are recommended.

The Technology Principle

According to NCTM (2000), "*Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning* [italics in original]" (p. 24).

The use of technology with gifted students can take two different avenues. First, there are tools that can be used to expand the understanding of the content. When research is conducted on mathematics and technology, most of the emphasis is placed on the use of calculators and computers for completing calculations or creating graphic models of problems and data. There seems to be some disagreement between those who believe in standard teaching approaches that have students wrestle with the calculations without the quick aid of technology and a more liberal group of people who feel that the principles of math are more evident if the emphasis is on the analysis and manipulation of data rather than the burden of computation. This debate will probably continue and instruction for the classroom will be based on the individual teacher's philosophy and approach to mathematical skill.

Both points of view have their place depending on the goal of the mathematical topic being presented at the time. However, this discussion falls far short of the second avenue that technological opportunities can offer to talented students. As teachers continue to teach technology natives, the application of technology to mathematics will become increasingly more demanding. Technology can be more than a tool or inspiration; it can also be an independent learning environment as long as the student is not isolated with the computer instruction.

As mentioned before, technology can offer opportunities for research on independent study of math topics, communication with mathematical experts, and actual real-time instruction for accelerated students. The Internet can provide a vast and exciting source of problems to solve and a way to communicate with other students of similar interests and abilities.

Although websites do not provide formal curriculum, they provide opportunities to investigate interesting, exciting, individualized explorations into math. The Internet can provide an individualized activity based on student need, interests, curiosity, and excitement for math. These opportunities can still be set up to parallel and build on the instruction offered in the basic core curriculum (Johnson, 1997). The following modified questions are helpful for evaluating whether a website is appropriate for high-ability learners (Johnson, 1997):

- Does the site contain a high level of sophisticated ideas? (Topics such as knots, infinity, or topology are examples.)

- Are there opportunities to extend the topic into multiple paths and challenges? (Students with a high rate of acquisition will consume resources more quickly than other students.)
- Can the content of the site be tailored to the needs of high-ability math learners? (The site should allow for different levels, interests, and backgrounds. Age and grade level should not limit activities, projects, and problems.)
- Does the site encourage higher order mathematical thinking skills? (Problem solving, math reasoning, research, exploration, and perseverance are just a few skills.)
- Does the information go beyond what the textbook offers? (If it duplicates textbook information, it does not support extension of mathematical talent.)

With video cameras, Skype, iChat, and other communication options, accelerated students can now be part of advanced classrooms that are miles away from their where they are located. They can participate in cross-district instruction or university instruction and not be isolated in their online options. Math instruction—indeed all instruction—is available in new and exciting formats for students. These options will only be effective if teachers and students are provided with the appropriate technology resources and support from computer experts.

The NCTM (2000) *Principles and Standards* and mathematical talent are certainly compatible. If each of the principles is interpreted with the abilities of *all* children in mind, they can be extremely effective. Mathematically talented students need to have accommodations made within their math journey that will keep enjoyment central to their experience. At times, they will also have to learn the satisfaction and value of struggling with concepts, of delving deeper into math content so that they can apply math theory to solving real problems. To that end, the authors offer the following suggestions for implementation of the standards for mathematically talented students.

At the administrative level:

- Districts should provide all teachers with opportunities to learn about the mathematically talented mind and gifted traits so that these students receive the appropriate accommodations that they need.
- Districts should also provide teachers with time to develop appropriate modified lessons to accommodate these traits.
- Teachers will need training on which students should be considered for an accelerated model in math and be trained on characteristics that indicate talented mathematical thinking.
- Elementary teachers will need content support and should be made aware of the scope of the standards for the next higher level. This will encourage awareness of the foundations for more complex thinking.
- Middle and high school math teachers who have more awareness of mathematical talent from their specialized field should consider alternate texts to encourage math talent.

At the instructor level, teachers have the major responsibility for keeping students enthralled with mathematical opportunities. There are a few tips to help manage this challenge more effectively. Teachers should:

- Organize the standard requirements by higher order skills and start with these requirements with talented students. Teachers then can cross into other subject areas such as research, technology, science, probability, statistics, and historical math studies for application.
- Try teaching the hardest concept first or introducing the problem before the concept. Those that grasp the concept quickly may then use the class instruction time for learning centers, web investigations, or independent projects related to math. Those that don't grasp the concept will use mathematical reasoning and exploration to intuitively solve the problem rather than use a rule-based algorithm.
- Develop a variety of models for introducing concepts rather than the sequential, spiral technique.
- Use a variety of text materials to help address the intent of the standards and not just the content. Relevant materials should build breadth, depth, and acceleration toward the next standard.
- Select online options that require more than one lesson. This provides time for the teacher to evaluate student reasoning and ability beyond the usual required calculations.

At the school level:

- Administration, teachers, and parents need to work as a team to creatively meet the needs of these students.
- Allow for flexibility with unusual schedule arrangements and delivery models.
- Consider authentic, alternative assessments to document talent.

Many other options for building on the standards were not touched on in this article. Exposure to community career opportunities that relate to the standards for each level is just one example. The opportunities for talented math students are only as limited as the imaginative solutions for delivery that teams can devise.

Mathematics is real life. It surrounds us and provides infinite fascinations, frustrations, and promises for the future. These students *need* support from all educators to find the harmony, beauty, and excitement in math that the standards strive to impart. **GCT**

Resources

Kettler, T., & Curliss, M. (2003). Mathematical acceleration in a mixed-ability classroom. *Gifted Child Today, 26*(1), 52–55, 65.

Miller, R. (1990). *Discovering mathematical talent*. Reston, VA: ERIC Clearinghouse on Handicapped and Gifted Children. (ERIC Document Reproduction Service No. ED321487)

Miller, R., & Deal, L. (2004). *Flexibly paced, differentiated instruction in mathematics for gifted students.* Unpublished document.

Wheatly, G. (2004). A mathematical curriculum for gifted and talented. In J. VanTassel-Baska (Ed.), *Curriculum for gifted & talented students: Essential readings in gifted education* (pp. 137–143). Thousand Oaks, CA: Corwin Press.

Ysseldyke, J. E., Tardrew, S., Betts, J., Thill, T., & Hannigan, E. (2004). Use of an instructional management system to enhance math instruction of gifted and talented students. *Journal for the Education of the Gifted, 27,* 293–310.

References

Assouline, S., & Lupkowski-Shoplik, A. (2003). *Developing math talent: A guide for educating gifted and advanced learners in math.* Waco, TX: Prufrock Press.

Berger, S. L. (1991). *Differentiating curriculum for gifted students*. Reston, VA: Council for Exceptional Children. (ERIC Document Reproduction Service No. ED342175)

Chuska, K. (2004). *Chuska Scale for Rate of Acquisition.* Retrieved from http://www.pagiftededucation.info/pdf/GiftedGuidelines.pdf

Colangelo, N., Assouline, S., & Gross, M. (2004). *A nation deceived: How schools hold back America's brightest students* (Vol. 1). Iowa City: The University of Iowa, The Connie Belin & Jacqueline N. Blank International Center for Gifted Education and Talent Development.

Davidson Institute for Talent Development. (2003). *Tips for teachers: Successful strategies for teaching gifted learners.* Retrieved from http://www.desarrollandomentes.com/curso/docs/Tips%20for%20Teachers%20of%20Gifted%20Learners.pdf

Heacox, D. (2002). *Differentiating instruction in the regular classroom*. Minneapolis, MN: Free Spirit.

Johnsen, S. (2005). Within-class acceleration. *Gifted Child Today, 28*(1), 5.

Johnson, D. T. (1997). Math curriculum extensions: Using the web. *Systems 5*(2). Retrieved from http://cfge.wm.edu/Documents/MathCurriculumExtensions.pdf

Johnson, D. T. (2000). *Teaching mathematics to gifted students in a mixed-ability classroom.* Reston, VA: Council for Exceptional Children. (ERIC Document Reproduction Service No. ED441302)

Lester, J. (2007). *Generating standards-based lessons: Planning and implementing lessons in the age of standards.* Marion, IL: Pieces of Learning.

Lockwood, A. (1992). The de facto curriculum. *Focus in Change, 6,* 8–11.

Loveless, T., & Coughlan, J. (2004). The arithmetic gap. *Educational Leadership, 61*(5), 55–59.

Maker, C. J. (1982). *Curriculum development for the gifted.* Rockville, MD: Aspen Systems.

Maker, C. J. (2004). Developing scope and sequence in curriculum. In J. VanTassel-Baska (Ed.), *Curriculum for gifted & talented students: Essential readings in gifted education* (pp. 25–40). Thousand Oaks, CA: Corwin Press.

Mann, E. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30,* 236–260.

National Association for Gifted Children. (2009). *2008–2009 state of the nation in gifted education*. Washington, DC: Author. Retrieved from http://www.nagc.org/uploadedFiles/Information_and_Resources/State_of_the_States_2008-2009/2008-09%20State%20of%20the%20Nation%20overview.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

No Child Left Behind Act, 20 U.S.C. §6301 (2001).

Pierce, R. (2002, April). A closer look at past and current research about mathematically talented children. *Our Gifted Children, 93,* 24–25.

Piirto, J. (2007). *Talented children and adults: Their development and education* (3rd ed.). Waco, TX: Prufrock Press.

Reys, B. J., Reys, R. E., & Chavez, O. (2004). Why mathematics textbooks matter. *Educational Leadership*, *61*(5), 61–66.

Roedell, W. C. (2000, February). Nurturing giftedness in young children. *Our Gifted Children*, *67,* 16–18.

Rotigel, J., & Fello, S. (2004). Mathematically gifted students: How can we meet their needs? *Gifted Child Today, 27*(4), 46–51.

Sousa, D. (2009). *How the gifted brain learns.* Thousand Oaks, CA: Corwin Press.

St. Cyr, S. (2004). Can distance learning meet the needs of gifted elementary math students? *Gifted Child Today, 27*(2), 42–50.

VanTassel-Baska, J. (2000). Standards of learning and gifted education: Goodness of fit. *Virginia Association for the Gifted Newsletter.* Retrieved from http://www.vagifted.org/newsletter.htm

VanTassel-Baska, J. (2009, November). *Using national teacher standards for NCATE recognition and for program development.* Paper presented at the annual meeting of the National Association for Gifted Children, St. Louis, MO.

VanTassel-Baska, J., & Little, C. (Eds.). (2003). *Content-based curriculum for high-ability learners.* Waco, TX: Prufrock Press.